**Critical points in the applications of geometrical optics to numerical simulation of lightning-related spectrograms**

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Numerical modeling of VLF spectrograms related to lightning-induced emissions is an efficient tool in studies of whistlers in the magnetosphere. However, most of work in this direction is based on intuitive ideas, but not on well-grounded concepts. This limits the numerically simulated spectrograms to an interesting and useful illustration of real ones. To make numerical simulation a powerful and reliable implement in the ionospheric and magnetospheric studies, the approach to numerical modeling should be well substantiated, and the frame of validity of the method should be clarified. The points that need to be explicated include: the wave field expansion into geometrical optics wave packets in an inhomogeneous medium; construction of frequency-time plots on spectrograms, i.e., finite-width curves on which the spectral intensity differs from zero; and, finally, determining the time-dependent spectral amplitude as a function of frequency and time from the wave packet energy density, with the account of the evolution of the latter in space and time. These and related questions are discussed in this report. A special attention is paid to the properties and the presentation of initial field, to the wave packet spreading during propagation, and to the choice of finite width of the frequency bin on a spectrogram in relation to the characteristic time of spectrum evolution. Since spectral intensity displayed in spectrograms is related to the amplitude of the wave electromagnetic field, while geometrical optics deals with the wave energy density, the relation between them is one of the key points in spectrogram modeling. While general expression for the wave energy density $U$ through the wave electric field amplitude and polarization coefficients is well known (e.g., *Shafranov*, 1967), as well as its explicit expression for parallel propagating whistler-mode waves (e.g., *Trakhtengerts and Rycroft*, 2008), the corresponding expression for the case of oblique propagation seems to be missing in the literature. Such an expression which will be presented in the report for the first time reads:

$$U= \frac{\left|E\_{x}\right|^{2}}{8π} \frac{ω\_{p}^{2} ω\_{c} cosθ}{ω\left(ω\_{c}-ω cosθ\right)^{2}}$$

where $E\_{x}$ is the magnitude of the wave electric field in *(****k****,* ***B***0*)* plane perpendicular to ***B***0, $ω\_{p} and ω\_{c }$are electron plasma and cyclotron frequencies, respectively, $ω$ is the wave frequency, and $θ$ is the wave normal angle, i.e., the angle between ***k*** and ***B***0.

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**References**

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