Solving multidimensional dispersion equation: phase analysis approach

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Root-finding is a common problem in various fields of physics and mathematics. This problem arises during the perturbation analysis of the system, resulting in a dispersion equation for the frequency ω and the wave vector $\mathbf{k}$: $f(\omega, \mathbf{k}) = 0$ where $\omega \in \mathbb{C}$ and $\mathbf{k} \in \mathbb{R}^n$, $n \geq 0$. Iterative Newton-like methods show high convergence rate, however, the continuity of the solutions in $\mathbb{C} \times \mathbb{R}^n$ is not used and the convergence of the method is strongly dependent on the input parameters such as an initial guess. If the function f is complicated enough, there can be no prior information about its zeros and, thus, no way to correctly pick the initial guess. The aim of the proposed algorithm is to find all zeros and poles inside a selected region without any information about the local behavior of the function f. The basis of the algorithm consists of a triangulation process and an implementation of the Cauchy's argument principle to inspect the constructed subregions. The refinement process is accompanied by an argument gradient $\nabla \arg(f)$ analysis that significantly decreases the number of points needed to correctly identify the locations of all zeros and poles.