**The relation between regression and optimal estimation method for nonlinear problems: the case for microwave measurements of the ozone.**

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In atmosphere remote sensing the two groups of retrieval methods are used: the first one require the minimization of some cost function for every retrieval result; the second produces the result by applying the explicit function to the data, while the function is the regression model obtained from a large prior ensemble of measurements and results pairs. The optimal estimation method (OEM, or maximum posterior distribution method) is most commonly used in the first group, more over many other methods (or more precisely the corresponding retrieval procedure) of this group can be formulated in terms of OEM even though they are based on entirely different concepts. The computationally expensive minimization implied by the methods of the first group significantly limits the complexity of forward model function used for near real time measurements. While the methods using regression models (e.g. retrials of tropospheric characteristics from microwave profiler data ) have no such problems, they somewhat lack the theoretical basis and often applied ad hoc.

It is known (see. the works of Clive. D. Rodgers) that in linear case OEM method is equivalent to multiple linear regression provided the prior ensemble is large enough. Non-linear cases are rarely explored theoretically (at least in remote sensing of the atmosphere) although they are of the most practical interest. In this work we show the relation between OEM and regression models in nonlinear case with large enough prior ensemble. In brief, considering the joint distribution of measurements and profiles (i.e. quantities to retrieve), OEM finds the maximum of the distribution given the measurement, while regression model tries to approximate the median of the said distribution. This fact is used in development of regression model algorithm for the problem of retrieval of middle atmosphere ozone vertical distribution from ground-based microwave radiometry data. The problem is well suited to illustrate the relation between methods as OEM is traditionally (in a great number of papers) applied to it, so it is done relatively easy, while the problem is nonlinear if one considers the estimation of tropospheric absorption part of the retrieval. The work of algorithm is demonstrated on simulated and real data.

The results allow us to produce regression based algorithms for various inverse problems with clear theoretical basis. One particular candidate is the problem of determining middle atmosphere temperature distribution using ground based microwave radiometry data where the complexity of forward model really hampers the real-time capabilities of the monitoring.