VARIATIONS OF CHORUS SOURCE LOCATION
DEDUCED FROM FLUCTUATIONS OF AMBIENT MAGNETIC FIELD: COMPARISON OF CLUSTER DATA AND THE BACKWARD-WAVE OSCILLATOR MODEL

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Abstract. We study the motion of the source region of magnetospheric chorus emissions using multi-point measurements of VLF wave emissions and geomagnetic field onboard the CLUSTER spacecraft. The geomagnetic field data are matched to a parameterized model of the local magnetic field, and the spatio-temporal dynamics of the magnetic field is obtained on this basis. The wave data from the WBD instrument are used to obtain the power-spectral density and number of chorus elements. Comparison of these data shows that chorus remains related to the magnetic field minimum, while the position of this minimum can vary rather strongly during the periods of enhanced geomagnetic activity. These results support the backward-wave oscillator (BWO) model of chorus emissions, which attributes chorus generation to an absolute instability of whistler-mode waves in presence of a step-like velocity distribution of energetic electrons. Such an instability takes place in the small vicinity of the local “magnetic equator” in a given magnetic field tube. Quantitative agreement between the data and the model is demonstrated by simultaneous variation of the statistical chorus characteristics and the deduced BWO parameters.

Introduction
Recently the multi-point measurements onboard the CLUSTER spacecraft opened new possibilities to study VLF wave emissions. The whistler-mode chorus emissions are clearly distinguishable thanks to their discrete nature and therefore it is possible to investigate the size and position of their generation region. Using measurements of the WBD instrument it was shown that the size of the chorus generation region is of the order of a few hundreds km perpendicular to the magnetic field line direction [Santolik and Gurnett, 2003]. The wave data from the STAFF instrument was used to obtain the energy flux in chorus waves, and the chorus source region was identified as the region where the energy flux is bi-directional. This region was shown to be extremely variable in space and time [Santolik et al., 2003, 2005].

The cyclotron interaction of the electromagnetic VLF waves with energetic electrons is most effective in a minimum of the magnetic field. Therefore the theoretical model of the VLF chorus generation [Trakhtengerts, 1999; Trakhtengerts et al., 2004] based on a backward-wave oscillator BWO analogy uses an idealized magnetic field in the equatorial plane of the magnetosphere. Taking into account the sensitivity of this region to the magnetospheric disturbances, the geometry of the local BWO should also vary. Here we try to answer the following questions: 1) Can we obtain the local BWO geometry directly from the satellite data? 2) Can we describe the dynamics of the BWO geometry during selected events?

The main characteristics of the magnetospheric BWO are: its position on the magnetic field line (position of the minimum of the field strength $B_{min}$ measured along the magnetic field line) and the effective length along the magnetic field line ($L_{BWO}$). To estimate these quantities we need to measure the magnetic field strength along a given magnetic field line. However, CLUSTER satellites give us values of the magnetic field in 4 points only. Therefore, a magnetospheric model is needed to compliment the measurements. The known global magnetospheric models are statistical, and, therefore, they cannot describe the dynamics of the observed events. Here we will use a dynamical model of local magnetic field constructed from the statistical model of Tsyganenko and Stern [1996] and additional currents to fit the evolution of the observed geomagnetic field. Previously, a similar approach but with one additional current was used to describe the dynamics of the magnetic field in the night sector at a radial distance of 4-10 Re during a substorm event observed by the CRRES satellite [Kozelov and Kozelova, 2003]. Here we use a more complex model with two additional currents.

Dynamical magnetic field model
The dynamical magnetic field model was constructed on the base of the statistical magnetospheric models Tsy-96 and Tsy-2004 [Tsyganenko and Stern, 1996; Tsyganenko and Sitnov, 2005]. We have then compared the output of this model to the magnetic field measured onboard the CLUSTER spacecraft by the FGM instruments[ Balogh et al., 2001]. Unfortunately, the deviations of the Tsy-2004 model from the locally observed values during the events of interest were too large. Therefore, we use Tsy-96 model with parameters adjusted for the least deviations from the local
The deviation of the observed magnetic field from value modeled by Tsyganenko-96 (Tsy-96) model:

$$\Delta B = B_{\text{observed}} - B_{\text{Tsy-96}}$$

We complete the model by 2 additional linear currents. The region of interest for us is located near the equatorial (in SM frame) plane, we therefore assume that each current is located at the parallel to the magnetic equator plane and the parameters of the currents are adjusted to fit the magnetic field values observed by two of the CLUSTER satellites.

For positions of two CLUSTER satellites, the magnetic field induced in vacuum by two currents:

$$\Delta B_{1,2} = \frac{\mu_0}{2\pi} \left( \frac{I_1 \times (p_{1,2} - r_1)}{||p_{1,2} - r_1||^2} + \frac{I_2 \times (p_{1,2} - r_2)}{||p_{1,2} - r_2||^2} \right)$$

where: $p_{1,2}$ are known positions of two CLUSTER satellites, $I_1$ and $I_2$ are two line currents, $i_1$ and $i_2$ are unit vectors along their directions, $r_1$ and $r_2$ are arbitrary points at the linear currents. To fix the currents in the planes in SM coordinates we should set values $r_{1z} = z_1$, $r_{2z} = z_2$, $i_{1z} = 0$, $i_{2z} = 0$ and relations:

$$r_{1z} = r_{2z} = p_{1z} + p_{2z}, \quad r_{1x} = p_{1x} + p_{2x}$$

Thus, we have only 6 scalar variables ($I_1$, $I_2$, $r_{1x}$, $r_{2x}$, $i_{1x}$, and $i_{2x}$) in 6 equations. The system of 6 equations has been analytically simplified in SM-coordinates to a system of 2 equations. The last system has been solved numerically for each time.

These additional currents may be interpreted as effective 'skin currents' at the boundaries of the plasma sheet during magnetospheric disturbances. The region of interest is located inside the region limited in z-direction by these currents. Therefore we do not try to extend our region of interest to distances beyond the interval $[z_1, z_2]$.

Then, the position of $|B_{\text{min}}|$ on a magnetic field line gives us the position of the local BWO. According to Trakhtengerts [1995] and Demekhov et al. [2003], the effective length of the local BWO can be estimated as the distance $L_{\text{BWO}} = |z_2 - z_1|$ between two points satisfying the following relations:

$$\Delta \varphi = \int \frac{dz}{v_i} \approx \frac{\omega_{\text{c}}}{v_i} \int b(z)dz = \pi$$

where $\Delta \omega = \omega_{\text{c}} - \omega_{\text{c0}}(1 + b(z))$, $b(z) = |B(z)/|B_{\text{min}}| - 1|$, $b(z_1) = b(z_2)$, and it is assumed that $\Delta = 0$ at the equator (where $\omega_{\text{c0}} = \omega_{\text{c}}$). $\omega_{\text{c}} = eB/m$ is the electron cyclotron frequency, $\omega_{\text{c0}}$ the equatorial electron cyclotron frequency.

Fig.1. Results of modeling of the magnetospheric BWO configurations on 18 April 2002: left column - for a magnetic field line at the CLUSTER-1 position; right column - at the CLUSTER-3 position. Top panels: dotted line is the observed strength of the magnetic field, dashed line - calculated by Tsy-96, solid line - fitted by a model with 2 additional currents. Second panels: the modeled strength of the magnetic field along the magnetic field line. Third panels: symbols - calculated positions of the magnetic field minimum, solid line - smoothed evolution of this position, dashed lines - positions of the additional currents, dotted line - the CLUSTER orbit. Bottom panels: evolution of the estimated length of the magnetospheric BWO.
Variations of chorus source location

Event on 18 April 2002

Figure 1 presents the results of modeling for the case on 18 April 2002 when the spacecrafts were closely separated. To fit the magnetic field configuration we use the data from CLUSTER 1 and CLUSTER 3 separated in the SM coordinate system by ~ 300 km along the Z-coordinate and by <50 km along the X and Y coordinates. In a contrast with the Tsy-96 model, the dynamical model gives the perfect fit to the observed values of the magnetic field in the near-equatorial region. Parameters of the magnetospheric BWO, its position at the magnetic field line and its effective length $L_{\text{BWO}}$, have been estimated using the obtained magnetic field. Both parameters exhibit strong fluctuations. Variation of the magnetospheric BWO position obtained as a position of the minimum B along the field line qualitatively agrees with the variation of the chorus source location previously obtained from the STAFF data [Santolik et al., 2005].

Event on 31 March 2001

On 31 March 2001 the CLUS TER spacecrafts were at relatively large separations. The measurements were done under very disturbed conditions, when the Kp index reached the value of 9 and Dst decreased to -358 nT [Baker et al., 2002]. To fit the magnetic field configuration we use the data from CLUSTER-1 and CLUSTER-2 separated in the SM coordinate system by ~1300 km along the Z-coordinate and by <150 km along the X and Y coordinates. The results are presented in Figure 2. The dynamical model describes the observed values of the magnetic field sufficiently well. Only a few times we can note a large deviation from the observed value, but beyond the main interval of interest (time interval from 07:05 to 07:12 UT) when the spacecraft were close to the equatorial region.

The obtained variation of the position of minimum B along the field line qualitatively agrees with the variation of the chorus source location previously obtained from the STAFF data [Santolik et al., 2005]. The estimated length of the BWO has step-like changes in the interval between 07:05 UT and 07:12 UT. These changes should modify the BWO generation characteristics. Some details of this peculiarity are shown in Figure 3. The threshold flux of the BWO generation was estimated from the BWO geometry and plasma parameters [Trakhtengerts et al., 2004]:

$$ S_{\text{thr}} = \frac{4\pi m_e c^2 v_{\text{res}}}{e h_{\text{step}} L_{\text{BWO}}^2 (1 + f_{h/2} / f)} v_{\text{rec}}^2, $$

where

$$ v_{\text{rec}} = \frac{\omega_{\text{rec}} - \omega}{k_{||}} = \frac{\omega_{\text{rec}} \sqrt{f / f_{h/2}}}{\omega_{\text{rec}} \sqrt{f / f_{h/2}}}, $$

is the parallel velocity of the resonant electrons. Here $\omega_{\text{pe}} \approx 5.64 \times 10^4 N_e^{1/2}$ is the electron plasma frequency, $N_e$ is the cold plasma density (for this event $N_e \approx 5 \text{ cm}^{-3}$), $h_{\text{step}} \approx 0.1$ is the relative height of the step in the distribution function of electrons on parallel velocity, $f_{h/2}$ is one half of the electron cyclotron frequency, $f$ is the lowest frequency of the chorus elements. We assume for estimates that chorus is generated by electrons with $v_{\perp} \approx v_{\text{rec}}$. 

Fig.2. The same as in Fig.1, but for 31 March 2001.
Three intervals of different values of the $S_{thr}$ parameter have been selected, see Fig.3. The average number of chorus elements (below than $f_{ch}$) and the average power (amplitude per minute) of chorus, as observed by the WBD instrument onboard CLUSTER [Gurnett et al. 2001] are presented in Table 1 for these intervals. One can see that these values are decreasing with the increase of the threshold flux $S_{thr}$.

![Figure 3. CLUSTER-1 observations on 31 March 2001. Top panel: power-spectrogram of the electric field. Middle panel: integral flux of electrons in two channels. Bottom panel: symbols are the estimated threshold flux ($cm^{-2}s^{-1}sr^{-1}$) for the BWO generation. Three time intervals are marked with solid lines A, B, and C.](image)

**Table 1.** Chorus characteristics in three intervals of CLUSTER-1 observations, see Figure 3.

<table>
<thead>
<tr>
<th>Interval, UT</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>07:05.5-</td>
<td>07:09-</td>
<td>07:11-</td>
</tr>
<tr>
<td></td>
<td>07:08</td>
<td>07:10.5</td>
<td>07:11</td>
</tr>
<tr>
<td>Average $S_{thr}$, $cm^{-2}s^{-1}sr^{-1}$</td>
<td>$10^7$</td>
<td>$2\times10^7$</td>
<td>$9\times10^7$</td>
</tr>
<tr>
<td>Average chorus power, mV m$^{-3}$ min$^{-1}$</td>
<td>36.4</td>
<td>17</td>
<td>5.6</td>
</tr>
<tr>
<td>Average number of elements in a minute</td>
<td>197</td>
<td>141</td>
<td>44</td>
</tr>
</tbody>
</table>

**Conclusions**

The magnetic field data from the CLUSTER spacecraft are matched to a parameterized model of the local magnetic field, and the spatio-temporal dynamics of the magnetic field is obtained on this basis. The derived position of the minimum of the magnetic field at the magnetic field line passing through the spacecraft position during the events of interest is found to move randomly. This motion corresponds qualitatively to the motion of the central position of the chorus source region derived previously from multicomponent measurements. The length and threshold flux ($L_{BWO}$, $S_{thr}$) of the magnetospheric BWO are estimated during the events of interest. The quantitative agreement between the data and the BWO model is demonstrated by correlation of the statistical chorus characteristics with the deduced BWO parameters.

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**References**


