ROLE OF THE FIELD-ALIGNED DENSITY DISTRIBUTION FOR EFFICIENCY OF ELECTRON SCATTERING BY HISS WAVES

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Abstract. In this paper we consider peculiarities of electron density distribution along field lines in the plasmasphere and corresponding effects of these peculiarities on relativistic electron interaction with whistler waves. We describe the approximation of field-aligned density distribution based on Interball-1 measurements. This approximation allows considering the shift of the plasma density minimum relative to the magnetic equator. We use this approximation to recalculate the diffusion rates related to the relativistic electron interaction with hiss waves. The shift of the plasma density minimum results in the two times increase of the pitch-angle diffusion rate for ~1MeV electrons.

Introduction

Energetic electron scattering and acceleration in the inner magnetosphere are mostly provided by the electron resonant interaction with whistler waves (Lyons and Williams, 1984; Trakhtengerts and Rycrof, 2008). The most intense whistler wave emission in the radiation belts corresponds to the lower band chorus waves (see Agapitov et al., 2013 and references therein). However, being intense in the outer radiation belt, chorus waves are almost absent in the plasmasphere L<4 where hiss waves play the major role in electron scattering (Meredith et al., 2007). Modern models of resonant interaction of hiss waves with relativistic electrons include many effects: multifrequency distributions of hiss (Meredith et al., 2007, Meredith et al., 2009), oblique propagation of hiss (Artemyev et al., 2013; Ni et al., 2013), variation of hiss amplitudes with the geomagnetic latitude (Mourenas et al., 2014), variation of the background plasma density along the field lines (Orlova et al., 2014). The latter effect is very important because the resonant conditions for hiss interaction with electrons are quite sensitive to the local plasma density.

To calculate diffusion rates and estimate the time-scale of electron scattering and acceleration several plasma density models are often considered: the global core plasma model (Gallagher et al., 2000), models based on CRESS observation (Sheeley et al., 2001; Meredith et al., 2003), the empirical models based on IMAGE RPI active sounding (Denton et al., 2006, Ozhogin et al., 2012). Only last models give approximations of a plasma density variation along field lines. However, these models are purely empirical and their parameters have no physical meaning. In this paper we use measurements of the plasma density by the ALPHA-3 experiment onboard the INTERBALL-1 spacecraft (Bezrukikh et al., 1998) to reproduce the fine distribution of the plasma density along the field lines by physical equations and test possible effects of this distribution on relativistic electron scattering by hiss waves.

1. Plasma density model

The ALPHA-3 experiment onboard the Interball-1 spacecraft measured ion spectra with the period varying from 18 s up to 2 min (Bezrukikh et al., 1998). Under assumptions of the Maxwell energy distribution and with taking into account effects of satellite potential, plasma corotation, and spacecraft motion an ion density amplitude can be restored (Kotova et al., 2002; Kotova, 2007). Due to the quasi neutrality condition the derived ion density can be supposed to be equal to the electron density. Thus, the empirical plasma density distribution can be obtained. Following Verigin et al. (2012) approach we use the approximation for plasma density along the field lines:

\[ N(\lambda, \Delta \lambda) = N_{eq} e^{\eta_{eq} q/(1-\eta_{eq})} \frac{1-\alpha_{load}}{1-\alpha_{load}} \sqrt{1-\eta_{eq} e^{\eta_{eq} q/(1-\eta_{eq})}} \]

(1)

where \( N_{eq} \) is an equatorial amplitude of the plasma density (in this study we derive it from (Sheeley et al., 2001) approximation),

\[ q(\lambda, \Delta \lambda) = \frac{m_p g R_E}{2 T_{el} L} \left( \frac{1}{\cos^2 \lambda_{eq}} - \frac{1}{\cos^2 (\lambda - \Delta \lambda)} + \frac{L}{3 L_R} \cos^6 \lambda_0 - \cos^6 (\lambda - \Delta \lambda) \right) \]

(2)

\[ \eta(\lambda, \Delta \lambda) = \sqrt{1+3 \sin^2 (\lambda - \Delta \lambda)} \frac{\cos^6 \lambda_0}{\cos^6 (\lambda - \Delta \lambda)} \]
\[ q_{eq} = q_{eq}(\lambda=\Delta \lambda), \quad \eta_{eq} = \eta_{eq}(\lambda=\Delta \lambda), \]
\[ m_p \text{ is the proton mass, } T_p \text{ is the proton temperature (in this study we use } T_p = 2110 \text{ C}), \]
\[ R_E \text{ is the Earth radius, } L \text{ is the shell number and } L_R = 5.78 \text{ is Roche limit,} \]
\[ \alpha_{\text{load}} \in [0,1] \text{ is a filling factor (see details in (Verigin et al., 2012)),} \]
\[ \cos^2 \lambda_p = \frac{R_E + h_{ex}}{LR_E} \]

and \( h_{ex} = 1000 \text{ km} \) is height of the exobase. Model profiles of the electron (or plasma) density along the field lines are shown in Fig. 1(a) with the approximations given by the IMAGE RPI experiment (Denton et al., 2006). One can see that model (1) has a weaker gradient of the plasma density in the vicinity of the equator. Additionally, model (1) allows to consider the shift of the plasma density minimum away from the equator \( (\Delta \lambda \neq 0) \). Statistics collected by the ALPHA-3 experiment indicate that \( \Delta \lambda \) can be as large as 15°. In the following section we consider the effect of such shift of the plasma density minimum from the equator.

**Figure 1.** The distribution of the plasma density along the field lines for different system parameters \( (L=2.5) \).

### ii. Wave-particle resonant interaction

To investigate the efficiency of the wave-particle resonant interaction we calculate the pitch-angle diffusion coefficients describing relativistic electron scattering by whistler waves (Trakhtengerts, 1966; Kennel and Petschek, 1966). The local (for fixed magnetic latitude) expression for pitch-angle diffusion coefficient \( D_{\alpha \alpha} \) can be written as (Glauert & Horne, 2005):

\[
D_{\alpha \alpha} = \sum_{n} \sum_{m} B_{\omega}^2(\omega_i) \frac{\sin^2 \alpha}{\omega_i^2} \frac{g(X)XdX}{4\pi N(\omega_i)^2} \frac{n\Omega_e/\gamma - \sin^2 \alpha}{\cos \alpha} \left( \frac{\Delta \omega}{\omega_i} \right)^2 \left( \frac{\omega - \omega_i}{\omega_i} \right) \left( -\frac{\partial \Phi_{n,k}}{\partial \omega} \right) \]

where \( X = \tan(\theta) \) and \( \theta \) is the angle between wavevector \( \mathbf{k} \) and a background magnetic field, \( \Omega_e \) is the electron local gyrofrequency, \( \alpha \) is the electron pitch-angle, \( \omega_i \) is the resonant frequency given by the resonance condition \( \omega_i = k \cos(\alpha) \cos(\theta) - n\Omega_e/\gamma \), \( v \) is the amplitude of electron velocity, and \( \gamma \) is the electron gamma factor. In Eq. (4) the index \( i \) denotes the resonance number, while \( n \) is the harmonic number. The function \( \Phi_{n,k} \) determines the relation between wave electric and magnetic fields for resonant waves (Lyons, 1974). To describe electron resonant interaction with hiss waves we use the simplified dispersion relation:

\[
\omega = \frac{\Omega_e \cos \theta}{1 + (\omega_{pe}/kc)^2} \]

where \( \omega_{pe} \) is the plasma frequency. Functions \( B_{\omega} \), \( g(X) \), and \( N(\omega) \) determine wave spectrum, \( \theta \) distribution and its normalization:

\[
B_{\omega} = \sum_A \exp \left( -\frac{(\omega - \omega_m)^2}{\delta \omega^2} \right), \quad g = \exp \left( -\frac{(X - X_m)^2}{\delta X^2} \right), \quad N = \frac{1}{2\pi^2} \int_{X_{\max}}^{X_{\max}} \frac{g(X)XdX}{1 + X^2} \frac{k^2 \partial k}{\delta \omega} \]

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In Eq. (6) the index \( s \) denotes the inputs from different wave populations with corresponding mean frequency \( \omega_{w0} \) and frequency dispersions \( \delta \omega \). In this paper we consider three wave populations with parameters and normalizations \( A_{i} \) given by Artemyev et al. (2013). The distribution of wave normal angles \( \theta \) is assumed to be the same for all three wave populations with the mean \( X_{m}=X_{0}(\lambda) \) and the dispersion \( \delta X_{m}(\lambda) \) provided by Cluster observations (see Artemyev et al., 2013). The diffusion time is much larger than the electron bounce oscillation period. Thus, we need average Eq. (4) over the bounce oscillations (Lyons, 1974):

\[
\langle D_{aa} \rangle = \frac{1}{T} \int_{0}^{\lambda_{\text{max}}} D_{aa} \frac{\cos \alpha}{\cos^{2} \alpha_{eq}} \cos^{2} \lambda d\lambda \tag{7}
\]

where \( \alpha_{eq} \) is an electron equatorial pitch-angle, \( T=T(\alpha_{eq}) \) is the period of bounce oscillations, \( \lambda_{\text{max}} \) is the latitude of the electron mirror point, the local pitch-angle \( \alpha \) can be defined form the conservation of the magnetic moment:

\[
\sin^{2} (\alpha) = \sin^{2} (\alpha_{eq}) B(\lambda)/B_{eq} \text{ and } B(\lambda) \text{ is the background dipole magnetic field with the equatorial value } B_{eq}.
\]

We consider electron scattering by whistler waves at \( L=2.5 \) where the plasma model gives \( \omega_{pe}/\Omega_{e}=3.1 \) at the equator (Sheeley et al., 2001). The distribution of \( \omega_{pe} \) along the field lines is taken from the model presented in section I. Fig. 2 shows \( \langle D_{aa} \rangle \) as functions of \( \alpha_{eq} \) for four energies of electrons and three sets of parameters of the density model. For different energies the effect of the symmetry of the plasma density distribution is more important at different pitch-angle ranges. The most pronounced effect can be observed for \( -1 \) Mev electrons: in the vicinity of loss-cone \( (\alpha_{eq}<11') \) pitch-angle rates increase two times for \( \Delta \lambda=15' \) in comparison with the symmetric distribution \( (\Delta \lambda=0) \). For very large energies (-5 MeV) the effect is opposite: the asymmetry of the density distribution results in the 30% decrease of the diffusion rates.

![Figure 2](image)

**Figure 2.** Pitch-angle diffusion rates \( (D_{aa}) \) is normalized on \( p^{2} \) with \( p=m_{e}(\gamma^{2}-1)^{1/2} \) for three sets of parameters and four energy values.

### III. Discussion&conclusions

The shift of the plasma density minimum relative to the geomagnetic equator results in the increase of the diffusion rate in the vicinity of the loss-cone for \( -1 \) MeV electrons. Although this increase seems to be small (only factor \( -2 \)) it can potentially play an important role for electron scattering in the plasmasphere. Lifetimes of 1 MeV electrons in this region is less than 10 days (Meredith et al., 2009), while numerical calculations give 10-20 days (Meredith et al., 2007). Thus, the decrease of lifetime due to peculiarities of the density distribution along the field lines can help to overcome this discrepancy. It is also interesting to note that observed effect of the density distribution on pitch-angle diffusion rate depends on the particle energy. For small energy electrons (0.5-2 MeV) we obtain the pronounc increase of the diffusion rate, while for high-energy electrons (-5 MeV) we found the decrease of the diffusion rate for the system with the shift of the density minimum.

One additional possible effect of peculiarities of the density distribution along the field lines corresponds to the electron scattering (and/or acceleration) by magnetosonic waves. This wave emission is very important for electron scattering in plasmasphere where magnetosonic waves can effectively interact with electrons. The electron scattering by magnetosonic waves can help to fill the gap in the profile of the diffusion rate versus pitch-angle (Meredith et al., 2009). The electron lifetime strongly depends on the minimum value of the diffusion rate and, thus, an additional increase of \( \langle D_{aa} \rangle \) around the local minimum can substantially influence the final electron lifetime (see discussion in Mourenas et al., 2013). Amplitude of diffusion rates provided by electron resonant interaction with magnetosonic waves is very sensitive to the equatorial amplitude of the plasma density (Shprits et al., 2013,
Mourenas et al., 2013). Thus, shift of the plasma density minimum should influence significantly on impact of magnetosonic waves on electron scattering.

In conclusion, in this paper we have demonstrated that the plasma distribution along the field lines can have minimum shifted relative to the geomagnetic equator. This shift results in a modification of electron resonant interaction with whistler waves in the Earth radiation belts at \( L \approx 2-3 \). We have found the corresponding increase of the pitch-angle diffusion rates in the vicinity of loss-cone for ~1MeV electrons.

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References


